Aalto University
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Scattering from a sphere of nonlinear material

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Contents

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- Nonlinearity generates harmonics

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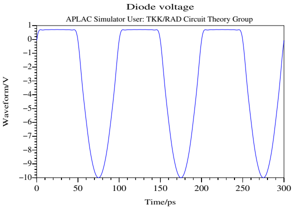
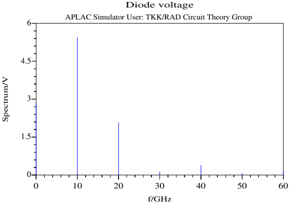
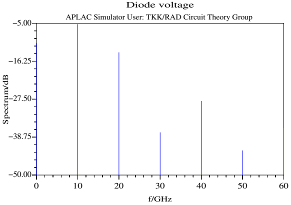
- Nonlinearity generates harmonics
  - How much energy is shifted from the fundamental frequency to the harmonics?
  - Type of nonlinearity used: that of a pn-junction (exp. function)
    - A real material; it has the most extreme nonlinearity possible



Waveform



Spectrum (V)



Spectrum (dB)

Why study scattering from a sphere of nonlinear material?

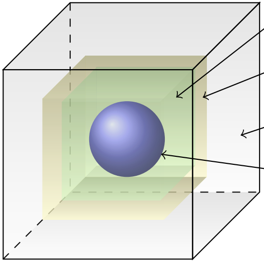
- Why sphere?
  - it is a canonical benchmark whose radar cross-section (RCS) is well known (Mie scattering)

Why study scattering from a sphere of nonlinear material?

- Why sphere?
  - it is a canonical benchmark whose radar cross-section (RCS) is well known (Mie scattering)
- Can nonlinear materials be exploited in stealth technology?

Model of the problem to be simulated

Problem is simulated using the FDTD method



green region: plane wave source (TFSF region)

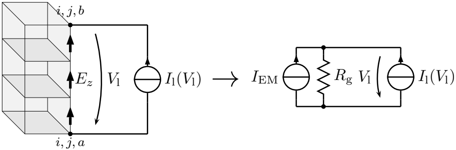
yellow surface: Huygens surface

grey surface: computational space terminated in CPML

sphere of nonlinear material

The Huygens surface is required to obtain the far field

Modelling the nonlinear material
Modelling a lumped element in FDTD



$$\oint_C \mathbf{H}^{n+\frac{1}{2}} \cdot d\mathbf{l} = \int_S \left(\varepsilon \frac{\partial \mathbf{E}^{n+\frac{1}{2}}}{\partial t} + \sigma \mathbf{E}^{n+\frac{1}{2}} + \mathbf{J}_\text{l}^{n+\frac{1}{2}} \right) \cdot d\mathbf{s}$$

Ampere's law

Update equation:

$$E_z\big|_{i,j,k+\frac{1}{2}}^{n+1} = \left(\frac{1 - \frac{\sigma \Delta_t}{2\varepsilon}}{1 + \frac{\sigma \Delta_t}{2\varepsilon}} \right) E_z\big|_{i,j,k+\frac{1}{2}}^{n} + \left(\frac{\frac{\Delta_t}{\varepsilon}}{1 + \frac{\sigma \Delta_t}{2\varepsilon}} \right) \times \left\{ \frac{\mathcal{I}_z\big|_{i,j,k}^{n}}{\Delta_x \Delta_y} - \frac{I_\text{l}(V_\text{l}^{n+\frac{1}{2}})}{\Delta_x \Delta_y} \right\}$$

$$\mathcal{I}_z\big|_{i,j,k}^{n} = (H_y\big|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y\big|_{i-\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}})\Delta_y - (H_x\big|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - H_x\big|_{i,j-\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}})\Delta_x$$

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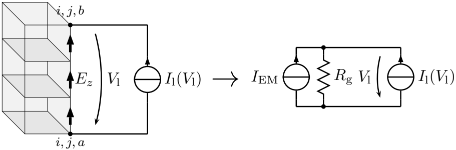

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Lumped elements can be directed only in the direction of the primary axes

Modelling the nonlinear material
Modelling a diode spanning one or many Yee cells

Diode current-voltage equation: $I_\mathsf{d} = I_\mathsf{s}\left(\mathrm{e}^{\alpha V_\mathsf{d}} - 1\right)$, $\alpha = q/(kT)$

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- Diode voltage V_d:
  - spanning one cell from i,j,a to $i,j,a+1$: $V_\mathsf{d}^{n+\frac{1}{2}} = -E_z\big|_{i,j,a}^{n+\frac{1}{2}}\Delta_z$
  - spanning cells from i,j,a to i,j,b: $V_\mathsf{d}^{n+\frac{1}{2}} = -\Delta_z \sum_{k=a}^{k=b-1} E_z\big|_{i,j,k}^{n+\frac{1}{2}}$

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- Iterate $V_\mathsf{d}^{n+\frac{1}{2}}$ using, e.g., Newton-Raphson:

$$V_\mathsf{d}^{n+\frac{1}{2},l+1} = V_\mathsf{d}^{n+\frac{1}{2},l} - \frac{V_\mathsf{d}^{n+\frac{1}{2},l}/R_\mathsf{g} + I_\mathsf{s}\left(\mathrm{e}^{\frac{q}{kT}V_\mathsf{d}^{n+\frac{1}{2},l}} - 1\right) - I_\mathsf{EM}}{1/R_\mathsf{g} + \frac{q}{kT}I_\mathsf{s}\mathrm{e}^{\frac{q}{kT}V_\mathsf{d}^{n+\frac{1}{2},l}}}, V_\mathsf{d}^{n+\frac{1}{2},0} = V_\mathsf{d}^{n-\frac{1}{2}}$$

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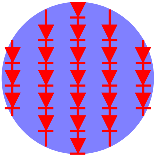
- Compute I_d and plug in $I_\mathsf{l} = -I_\mathsf{d}$ via the update equations in the column of cells in which it exists

Sphere of nonlinear material

- Sphere is filled with z-directed diodes
- A diode current passes through every Yee cell within the sphere



Encountered problems and work-arounds

- The presence of many diodes in close proximity makes the simulation susceptible to instability
  - the material is strongly nonlinear
- The time step may have to be reduced from the largest possible step obtained via the Courant-Friedrichs-Lewy condition
- The plane-wave amplitude cannot be arbitarily large
  - the strong nonlinearity in the current-voltage dependence causes instability
    - the larger the field strength, the larger the voltage implying larger changes in current even for relatively small changes in the large voltage

Simulations

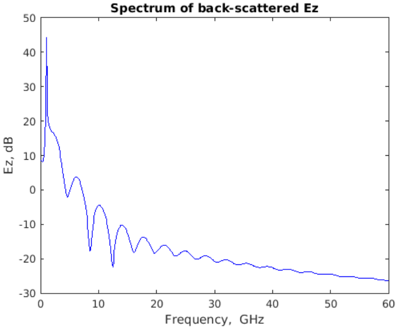
- Different spheres are simulated for
  - wavelength much larger than the sphere
  - wavelength of similar size as the sphere

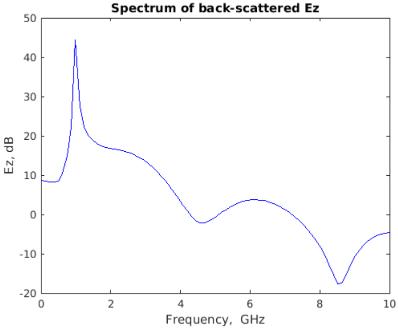
Simulation parameters

- Grid size: $120 \times 120 \times 120$
- Discretisation: $\Delta_x = \Delta_y = \Delta_z = 0.5$ mm, time step damping factor: $0.5 \Rightarrow \Delta_t = 0.963$ ps
- Plane wave propagates along x-axis with amplitude 5000 V/m
- Diode saturation current: 1 pA (typical value for a silicon diode)
- Sphere diameter: 32 mm (64 cells at equatorial plane)
- Scattered-field observation point: $15, 60, 60$ (between CPML and the Huygens surface)

Simulations: first a PEC sphere

Case 1a: $f = 1$ GHz, $\lambda \approx 300$ mm, time steps $n = 16\,384$





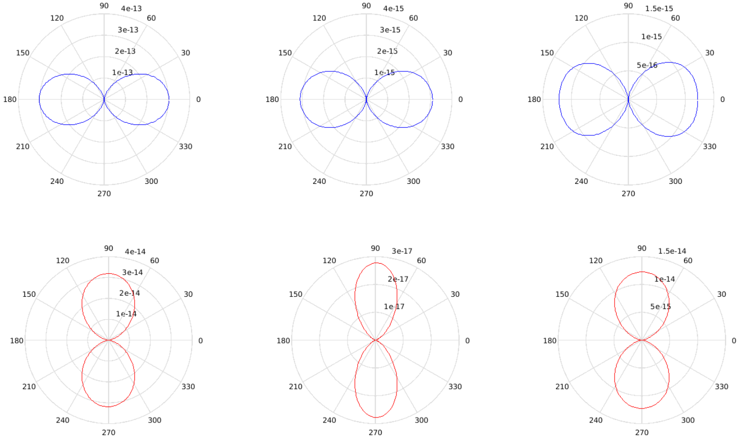
- The plane-wave frequency is obvious.
- The model errors are also obvious
  - the error can be reduced but not eliminated by increasing n

Simulations: first a PEC sphere

Case 1a: $f = 1$ GHz, $\lambda \approx 300$ mm (arbitrary scale)

RCS$_\theta$

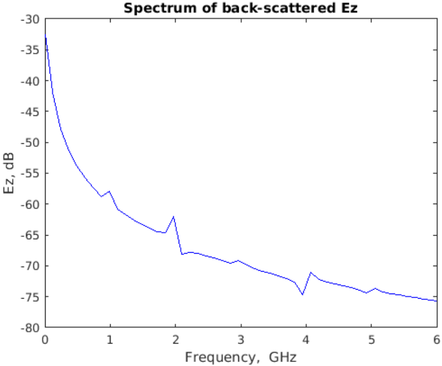
RCS$_\phi$



f $\quad$ $2f$ $\quad$ $3f$

Simulations: diode sphere

Case 1b: $f = 1\,\mathrm{GHz}$, $\lambda \approx 300\,\mathrm{mm}$, $n = 16\,384$, $\varepsilon_\mathrm{r} = 1$

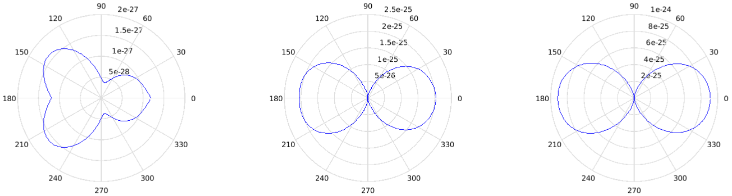


- A very strong DC component (for electromagnetic rectification) is present
- Harmonic peaks are distinguishable

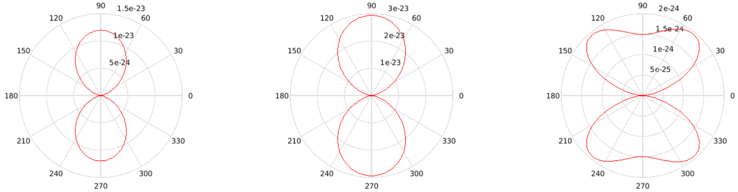
Simulations: diode sphere

Case 1b: $f = 1$ GHz, $\lambda \approx 300$ mm, $\varepsilon_\mathrm{r} = 1$ (arbitrary scale)

RCS$_\theta$



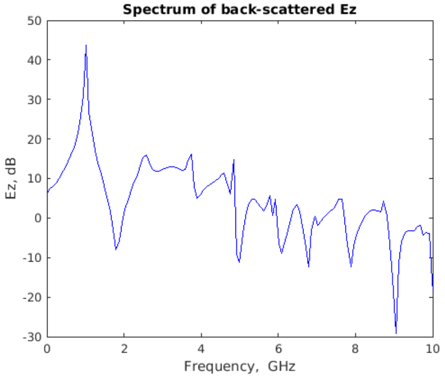
RCS$_\phi$



f $2f$ $3f$

Simulations: diode sphere

Case 1c: $f = 1\,\text{GHz}$, $\lambda \approx 300\,\text{mm}$, $n = 16\,384$, $\varepsilon_\text{r} = 11.68$



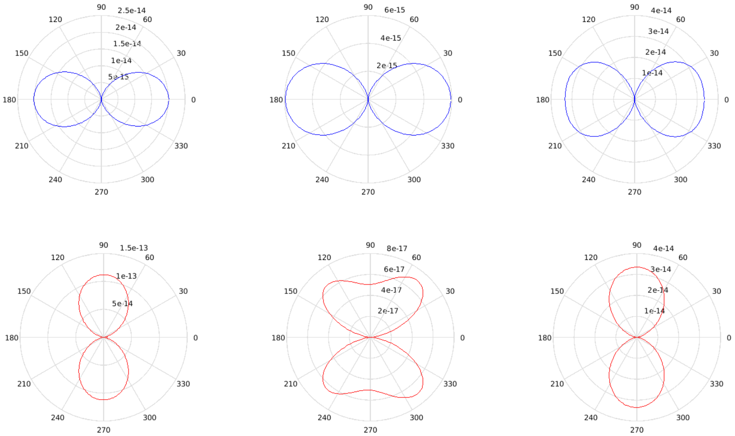
- The fundamental frequency is clearly visible

Simulations: diode sphere

Case 1c: $f = 1$ GHz, $\lambda \approx 300$ mm, $\varepsilon_r = 11.68$ (arbitrary scale)

RCS_θ

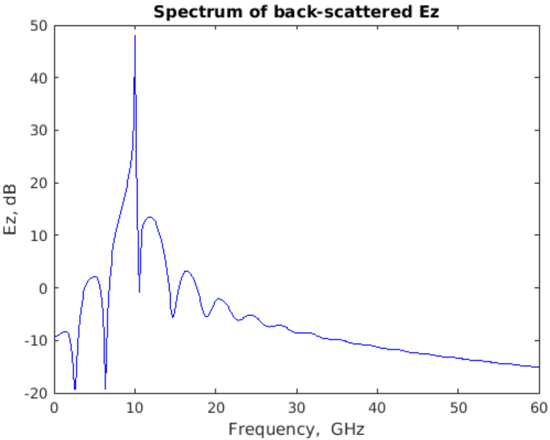
RCS_ϕ



f $\quad$ $2f$ $\quad$ $3f$

Simulations: again a PEC sphere

Case 2a: $f = 10\,\text{GHz}$, $\lambda \approx 30\,\text{mm}$, $n = 16\,384$

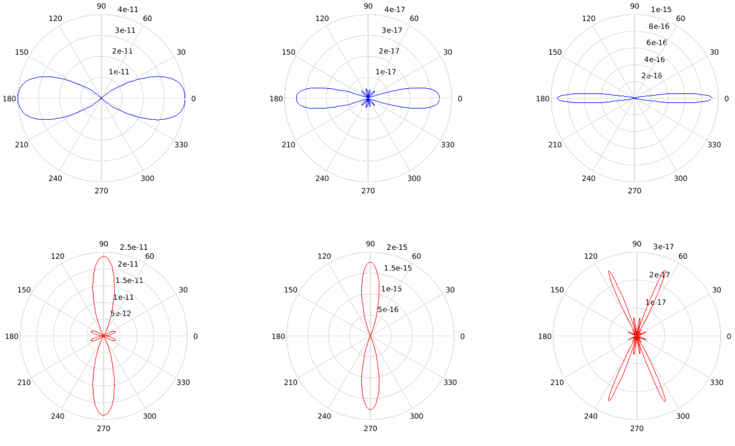


The plane-wave frequency and model error are obvious here too.

Simulations: PEC sphere
Case 2a: $f = 10\,\text{GHz}$, $\lambda \approx 30\,\text{mm}$ (arbitrary scale)

RCS$_\theta$

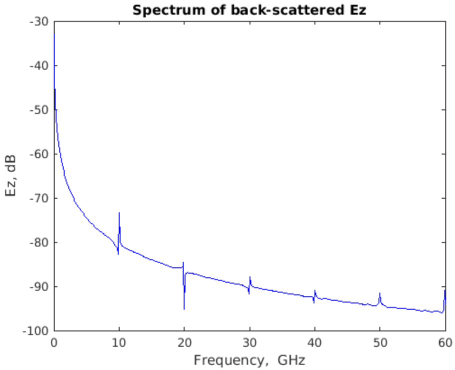
RCS$_\phi$



f $\quad$ $2f$ $\quad$ $3f$

Simulations: diode sphere

Case 2b: $f = 10\,\text{GHz}$, $\lambda \approx 30\,\text{mm}$, $n = 16\,834$, $\varepsilon_\text{r} = 1$



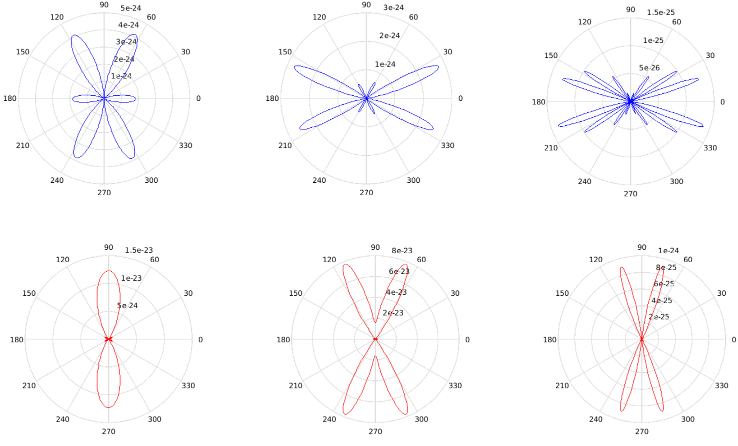
- A very strong DC component is present
- The fundamental frequency and harmonics are visible

Simulations: diode sphere

Case 2b: $f = 10\,\text{GHz}$, $\lambda \approx 30\,\text{mm}$, $\varepsilon_\text{r} = 1$ (arbitrary scale)

RCS_θ

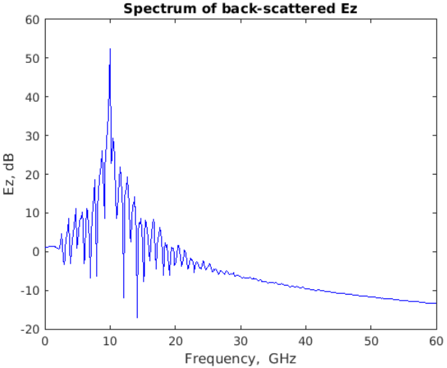
RCS_ϕ



f $2f$ $3f$

Simulations: diode sphere

Case 2c: $f = 10\,\mathrm{GHz}$, $\lambda \approx 30\,\mathrm{mm}$, $n = 16\,834$, $\varepsilon_\mathrm{r} = 11.68$



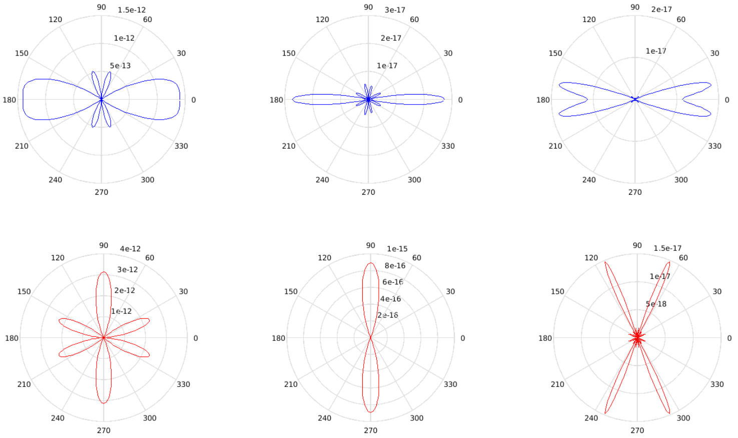
- The fundamental frequency is clearly visible

Simulations: diode sphere

Case 2c: $f = 10$ GHz, $\lambda \approx 30$ mm, $\varepsilon_r = 11.68$ (arbitrary scale)

RCS$_\theta$

RCS$_\phi$



f $2f$ $3f$

What do the data tell us?

$\varepsilon_r = 1$

- Harmonics are generated as expected, but they are weak
- Charge accumulates at the nodes of the diodes since they have no discharge path available
  - the scattered field stays positive in the back-scattered direction
  - a prominent DC component emerges in the spectra as a result

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$\varepsilon_r = 11.68$

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Other remarks

- Several cycles of the plane wave are required for the nonlinearities to kick in
- A relatively wide modulated pulse is necessary for the nonlinearities to have an effect

What next?

- Is it possible to arrange the diodes so as to enhance the effects of nonlinearities?
- Can the incident polarisation current be manipulated through a suitable arrangement of the diodes?
- Study how different pulses interact with spheres of such nonlinear materials

Conclusions

- A model for a nonlinear material is successfully implemented and used to compute scattering from a sphere made of a nonlinear material
- Simulation results show interesting behaviour in the sphere modelled
  - harmonics are created, but not always
- Energy is shifted primarily to DC, but to other frequencies as well
- Application to stealth technology requires further study