



## TIIVISTELMÄRAPORTTI (SUMMARY REPORT)

### Modern Signal Processing Methods in Passive Acoustic Surveillance

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#### Abstract

The problem studied in this research is the passive estimation of the direction of arrival (DOA) of several acoustic sources, using an array of sensors. The main application of this work is in underwater passive surveillance. Our focus is on *robust* DOA estimation, where the positions of the sensors are uncertain or affected by errors, with a known bound on the magnitude of the uncertainties or errors. This is the typical case of an array towed by an unmanned underwater vehicle (UUV). Moreover, we search estimation methods with low complexity, similar to that of minimum variance distortion-less response (MVDR), due to the possible computing power limitations of an UUV. We obtain such a DOA estimation method by posing the robust DOA problem such that the optimization is performed in two stages. First, the problem is relaxed and the corresponding power estimation has an expression similar to that of standard beamforming. If the relaxed solution does not satisfy the magnitude bound, an approximation is made by projection. Unlike other robust DOA methods, no eigenvalue decomposition is necessary and the complexity is similar to that of MVDR. The basic method is described for sinusoidal sources with known frequency and can be extended to the case of general sources by splitting the sensor signals on frequency bands and then summing over frequencies or transforming on a single frequency band. For low and medium SNR, the proposed method competes well with more complex methods and is clearly better than MVDR.

## 1. Introduction

Using sensor arrays for the passive estimation of the direction of arrival (DOA) of underwater sounds is an old technique, but new algorithms are proposed regularly for reformulations of the basic problem. We investigate robust DOA estimation, where the positions of the sensors are uncertain or affected by errors. This way of posing the problem is useful in practice in several situations, for example for coping with calibration errors. However, the most typical case is that of arrays towed by an unmanned underwater vehicle (UUV), since the sensors are often not placed on a rigid frame and movement in water induces variations of the relative positions of the sensors. It is commonly assumed that the position errors have known magnitude bounds, which come from the geometry of the physical array and from a study of its typical movement.

There are at least two general ways of enforcing robustness. They operate through the steering vectors that describe the relation between the geometry of the array and the position of a far-field source. In the robustness problem, the nominal steering vector that corresponds to the assumed position of the sensors (or a "central" position, in case of movement) is replaced by a set of vectors around the nominal one.

Worst-case robustness ensures an optimal beamforming quality for all these steering vectors. Methods belonging to this category can be found, among others in [VGL03,LoBo05]. They often are solved using convex optimization, see for example [Gersh10]. The advantage is that several efficient libraries are available for solving the problems. However, the complexity is often large, despite the recent advances in convex optimization algo-



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rithms. One can also argue that the approach may be too conservative. Aiming to ensure good enough quality for all possible steering vectors flattens the peaks of the DOA spectrum and makes more difficult the estimation of the number of sources.

A second way of ensuring robustness is to find the steering vector in the vicinity of the nominal one that maximizes the estimation of the power on the current direction, for the data at hand. Examples of such approaches are [LSW03,LSW04]. This leads to less complex methods and a sharper DOA spectrum. However, a possible drawback is a higher influence of the noise on the positions of the peaks. This is the category of methods to which we contribute in this report.

There are also other types of methods, like the maximally robust Capon beamformer [RuPe13]. The recent review [Voro13] lists several other methods.

All the above work is done for the narrowband case. The extension to wideband is simple in principle and can be done with two methods. For both, the sensor signals are decomposed via DFT on frequency bands. The summation method independently estimates the powers for the desired angles and then sums (or averages) the results. The coherent transformations method uses diagonal focusing matrices that, for each frequency band and angle, project the sensor signals on a single central band; then, a unique covariance matrix is obtained and a narrowband DOA method can be applied. Despite the known principle, only very recently robust wideband methods were investigated in [Soma13].

## 2. Research objectives and accomplishment plan

The main objective was to find a robust DOA estimation method that has a complexity similar to MVDR and performance similar to that of more complex methods, especially in the practical conditions of low and medium SNR. The method should work for both narrow- and wideband sources.

## 3. Materials and methods

The reference methods that we used for robust DOA estimation are robust Capon beamforming [LSW03], denoted here MVDR-R, and doubly constrained Capon beamforming [LSW04], denoted here MVDR-DC. Both are designed for narrowband sources generating sinusoidal signals. Both use an optimization criterion inspired from that of the standard MVDR, depending on a steering vector constrained to a set around the nominal vector. Figure 1 depicts these sets for several methods. In MVDR-R, the steering vectors are in a ball around the nominal one; the constant  $\rho$  defines the maximum allowable distance from the nominal vector. However, since the optimization criterion is proportional to the norm of the vectors, only the inner part of the ball is active, for vectors whose norm is less than the norm of the nominal vector; this is shown in red in the figure. However, since position errors should not change the norm of a steering vector, MVDR-DC works with only the vectors with the same norm as the nominal vector, as shown in green in Figure 1.

Both optimization problems can be solved with specialized procedures, based on the Lagrange multiplier approach, that need only finding the unique solution of a nonlinear equation with a single variable. However, the eigenvalue decomposition of the empirical covariance matrix of the sensor signals is necessary. So, the complexity is clearly higher than that of MVDR. This is especially true in the wideband case, where many eigenvalue decompositions are used, one for each frequency band (in the summation method) and one for each angle (in the transformation method).

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MVDR-R and MVDR-DC give much better DOA estimations than MVDR, usually with a slight advantage for MVDR-DC.

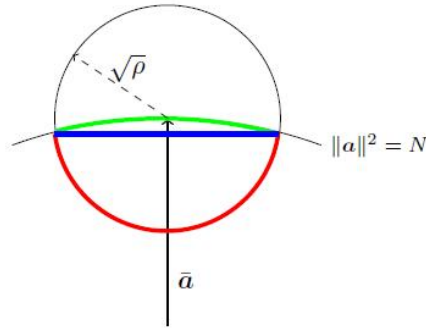


Figure 1 Sets of steering vectors considered by the optimization problem in MVDR-R (red), MVDR-DC (green) and our RAB method (blue).

Our proposed method, named robust adjusted beamforming (RAB) due to the resemblance of its initial DOA estimation to that produced by the standard beamforming, allows the steering vectors to lie on a hyperplane orthogonal on the nominal vector, respecting also the distance constraint. In Figure 1, this set is represented in blue. This is the nearest convex problem to that used in MVDR-DC (which is not convex). However, this is not its main advantage.

The RAB method has two steps:

1. The optimization problem is solved in a relaxed version, without the bound constraint; the steering vectors can lie on the whole hyperplane. The solution can be found analytically and implies only a matrix-vector multiplication and some less complex operations. If the bound constraint is satisfied, then the optimal DOA estimation is easily found; a norm correction is implicitly applied (such that the obtained vector has the same norm as the nominal one).
2. If the bound constraint is not satisfied, then we compute a projection of the obtained steering vector towards the nominal vector, such that the bound constraint is satisfied. With the projected vector, we compute an MVDR-like DOA estimation. This implies computing the inverse of the sensor signals covariance matrix, an operation that is necessary for all the other algorithms in the MVDR family.

So, the overall computation costs are only slightly higher than for MVDR. This remark applies also for the wideband case.

In the wideband case, a particular problem is the choice of the bound  $\rho$  for the different frequency bands, see [Soma13] for a discussion. We chose a linear dependence with frequency; the user has to give a single bound value, for the Nyquist frequency (this depends on the maximum magnitude of the possible variations of the sensor positions). For the other frequencies, the bound is thus automatically computed.

#### 4. Results and discussion

The methods have been tested via simulations; we give here only a few representative

results. We consider a uniform linear array (ULA) with  $N=12$  sensors; the distance between two consecutive sensors is  $d=0.25$  m. We assume an underwater DOA problem, the speed of sound being 1450 m/s. The sampling frequency is 8 KHz.

Since we assume that the actual sensor positions are uncertain, we perturb the nominal ULA positions with white Gaussian noise with standard deviation  $0.2 d$ , on both coordinates. Using these values, it results that in the worst-case, when the displacement is orthogonal on the direction of a source, the modification of the squared norm of the steering vector is about  $0.18 N$ . We chose the covering value  $\rho=0.3 N$  for the robustness bound.

Besides the described methods, we also give the results of the "oracle" MVDR, denoted MVDR-O, which is fed with the exact sensor positions. All the other methods work with the nominal positions.

*Narrowband results.* We simulate three sources, whose DOAs are 50, 100 and 130 degrees and whose amplitudes are 2, 1, 3, respectively. Their frequency is 2000 Hz.

RAB has the following typical behavior. In the vicinity of the true DOAs, the solution obtained at step 1 is valid, hence there is no need for further search. In the other, less relevant directions, the solution is no longer valid, hence the approximation described by step 2 has to be computed. This behavior is especially seen for medium and low SNR and an example is shown in Figure 2, for an SNR of 10 dB. One can see that RAB, MVDR-R and MVDR-DC give quite similar estimations, consistent with those of MVDR-O, despite the significantly lower complexity of RAB. The peaks of the robust methods are wider, and so the ability to separate close sources is diminished, but this is a natural price to pay. The standard MVDR can still give fairly good DOA estimation, but the powers of the sources are badly estimated. For lower SNR this relative behavior is somewhat similar, although the differences between the methods are smaller, due to the prevalence of the sensor measurement noise over the "noise" due to unknown sensor positions. However, robust methods are clearly better than standard MVDR.

When the SNR is high (more than 20 dB), RAB starts misestimating some peaks of the power. Its estimation quality may become lower than that of MVDR-R or MVDR-DC, but RAB is still better than MVDR.

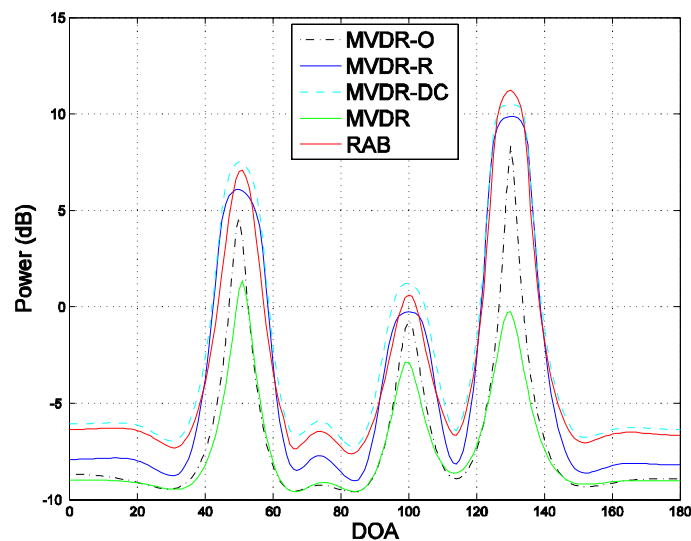


Figure 2 Typical DOA estimations in the narrowband case, SNR=10 dB.

*Wideband results.* We simulated two sources corresponding to AR processes whose spectrum is decaying as frequency grows, as common for ships, see [Hel13] for details. They are situated at angles 50 and 90 and their amplitudes are 2 and 1, respectively. We use an FFT of size 128 and 62 snapshots per frame, which means about one second per frame. We use the frequency bands from 500 to 3000 Hz; low frequencies are avoided for their low potential of discrimination and high frequencies for their noisy character. The central frequency for the transformation method is 2000 Hz.

The results of the summation and transformation methods are similar, so we report results only for the former. Figures 3 and 4 give examples of power estimation for a single frame, at SNRs of 10 and 0 dB, respectively. Again, the robust methods give better estimates of the relative amplitudes of the peaks and are close to MVDR-O. RAB appears quite reliable and gives good estimates at a cost significantly lower than MVDR-R or MVDR-DC.

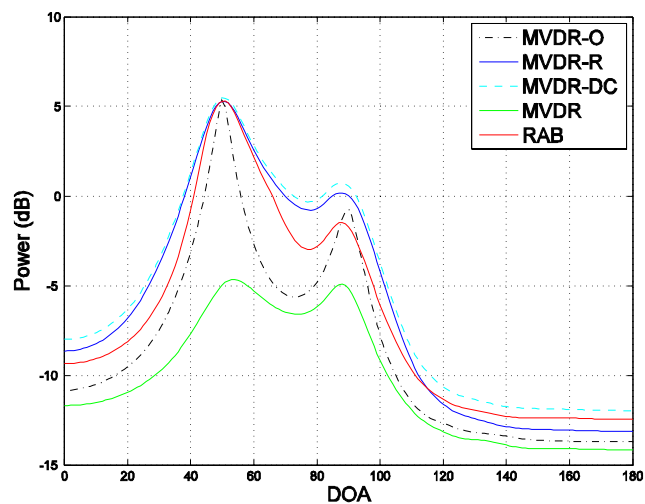


Figure 3. Wideband sources, SNR=10dB.

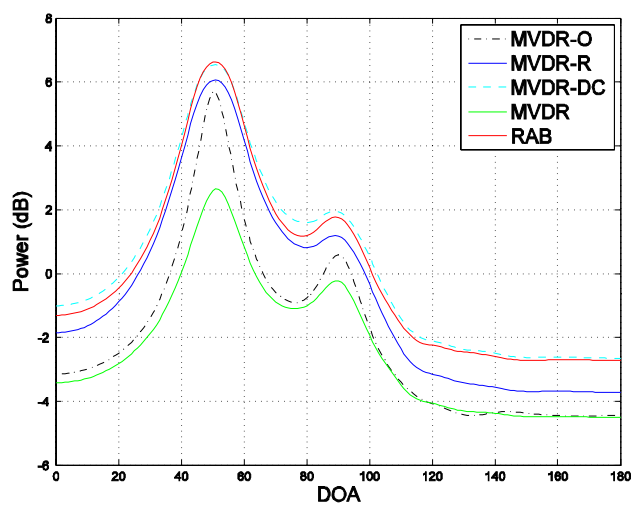


Figure 4 Wideband sources, SNR=0dB.

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## 5. Conclusions

We studied robust DOA methods for arrays whose sensors positions are affected by errors. In this case, robust methods give better estimations than standard MVDR, usually with a higher computational complexity. We have proposed a new robust method, called RAB (robust adjusted beamforming), based on an optimization problem that is slightly different from other MVDR-inspired methods, like robust or doubly constrained Capon beamforming. Unlike the other optimization problems, ours allows an easy-to-compute approximate solution, with a cost similar to that of MVDR. Simulations with narrow- and wideband sources show that RAB performs similarly to other robust methods at low and medium SNR, despite its lower complexity.

## 6. Scientific publishing and other reports produced by the research project

The following publication was submitted:

B. Dumitrescu, C. Rusu, I. Tabus, J. Astola, Low-complexity robust DOA estimation, submitted at ICASSP 2015.

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