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TIIVISTELMÄRAPORTTI (SUMMARY REPORT)

Military communication using orbital angular momentum based radio (MORAMBA)

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Abstract

The major weakness of any wireless communication system is the ease of intercepting or jamming radio signals, which increases the risk of using or altering radio signals by unauthorized persons or prevents the usability of a communication system. In this work, we propose to study the applicability of a novel radio transmission technique that exploits orbital angular momentum (OAM) state of radio beam to securely transmit information in military systems. In particular, we will develop methods to generate, receive, detect, and possibly intercept the OAM-based radio transmission. Research results can be used to design critical communication systems, for example, emergency services or civil protection, such that they will not be susceptible to electronic attacks or cyber-attacks. In Linturi et al. report, OAM-based transmission has recently been recognized as one of the 50 most important future technologies for Finland.

1. Introduction

The broadcast nature of the wireless communication medium makes it hard to eliminate unauthorized access to wireless networks. Hence, it is relatively easy to eavesdrop on it in general. Furthermore, in wireless networks, the risk of using or altering radio signals by unauthorized persons or preventing the usability of a communication system is significantly higher than in wired networks. Given the prevalence of wireless technologies, protection of own information transmission and recognition and, possibly, jamming of the enemy communication is of paramount importance.

In conventional military communication systems, the protection of own information is usually achieved by obscuring the information transmission using cryptography or special modulation schemes such as spread spectrum modulation, or both [Nicholson 1988]. However, in wireless military communications cryptography is not sufficient as it only converts the readable messages into guised unreadable information and thus does not provide any protection against eavesdropping or jamming. In spread-spectrum communications the signal is spread in accordance to a pseudorandom code over a frequency band in excess of the minimum bandwidth required to send it. At the receiver, the received signal is correlated with the pseudorandom code and the original transmitted signal is reconstructed. The two most common spread spectrum techniques are direct sequence and frequency hopping. In general, spread spectrum communication systems make eavesdropping, direction finding, and jamming difficult. For that reason, spread-spectrum modulation is considered a key technique of electronic counter countermeasures (ECCM) [Ziemer & Peterson 1985].

The ultimate purpose of jamming is to prevent the usability of the communications transmission [Nicholson 1988, Ziemer & Peterson 1985]. Jamming is performed by transmitting a signal, usually a band-limited noise, to the receiving antenna at the same frequency band as the original transmitter. In digital communications, jamming is successful when the error rate of the transmission cannot be compensated by the error correction techniques. This goal can be achieved either by jamming the synchronization signal so that the synchronization between transmitter and receiver is lost or by injecting an interference signal into a given frequency band so that the actual signal is completely submerged by the interference. Depending on the situation, some other jamming strategies might be more effective. Waveforms useful for jamming include but are not limited to noise-modulated frequency modulation, noise burst, continuous-wave tones, and swept signals [Ziemer & Peterson 1985]. However, for the jammer to be most effective, the jamming signal must be tailored to the communication system and to the actual received signal power. A jammer which has knowledge of the type of signalling being used, which can accurately predict the received signal power, and which can adapt to transmit the optimum jamming signal is called a smart jammer [Ziemer & Peterson 1985]. The field of study that includes the design and analysis of jammers and jamming strategies is called electronic countermeasures (ECM) [Ziemer & Peterson 1985].

The possibility of creating highly secure communications links impervious to threats and external attacks by exploiting light orbital angular momentum (OAM) has already been recognized by Defense Advanced Research Projects Agency (DARPA). In 2011, DARPA funded research efforts of Prof. Ramachandran from Boston University and Dr. Steve Golowich from MIT Lincoln Laboratory related to secure communication using optical vortices. The researchers were to investigate the properties of light beams carrying OAM in optical fibers and their applicability to creating next generation secure quantum encryption links, by encoding information in different angular momentum states [ECE News].

In [Linturi et al. 2013], OAM-based transmission in radio frequency bands has recently been recognized as one of the 50 most important future technologies for Finland.

2. Research objectives and accomplishment plan

In this work, we study the applicability of a novel radio transmission technique that exploits orbital angular momentum (OAM) state of radio beam to securely transmit information in wireless military systems. In particular, we study the possibilities of creating highly secure communication links using radio vortices rather than optical vortices.

The objectives of this research are: 1) to exploit the characteristics of signals transmitted using OAM-based radio techniques and identify the military communication systems, where OAM-based radio techniques can be implemented, 2) to examine possible limitations of the theory of OAM-based radio transmission suggested by the theory based on spherical wave functions, 3) to carry out theoretical and simulation based performance evaluation of OAM-based radio wave generation and reception techniques, taking into account realistic radio-electrical environments and relevant propagation parameters. Develop the suitable algorithms, 4) demonstrate by simulations the jamming resilience and difficulty of intercepting the signal transmitted with selected OAM-based radio techniques.

The research is divided into three phases: 1) System requirements and specification, 2) Algorithm development, 3) Numerical simulations.

In the first phase, the system requirements are studied. System requirements include performance and complexity requirements in terms of the efficiency of the use of materials, energy, control information, and security metrics. The target system is the system of the whole project. We emphasize clear scenario and problem descriptions, initial algorithm design, and system design plan. The problem description takes into account all the things we have learned about complexity and networks, radio beam generation and reception using multiple antennas, and optimal jamming strategies. The scenario descriptions, on the other hand, take into account specific needs of military communications in the battlefield and unique properties of OAM-based radios.

In the second phase, to be executed in the year 2014, new radio transmission schemes will be developed. In “wave-on-waves” experiment a specially crafted parabolic antenna, also known as spiral-phase-plate antenna, was used to generate radio beam carrying OAM. In this work, we will study and develop alternative ways to generate and receive radio beams carrying OAM using, for example, planar, circular, or spiral antenna arrays. The properties and interception possibilities of radio beams carrying OAM will be studied. New signal processing algorithms for controlling antenna arrays will be developed.

Finally, in the third phase to be executed in the year 2015, the high-level algorithm simulations will be made in MATLAB and commercial electromagnetic field simulation software. Visualization tools will be developed to help to understand the properties of the radio beams carrying OAM as well as generation, reception, detection, and interception of those radio beams.

3. Materials and methods

Any electromagnetic system radiates energy, commonly referred to as linear momentum (LM), and angular momentum into far zone. Electromagnetic waves carrying angular momentum were first theoretically studied by Poynting in 1909 [Poynting 1909]. The law of conserving angular momentum was verified experimentally by Beth [Beth 1935] in optical waveband in 1935 and in the centimetre wavelength by Carrara in 1949 [Carrara 1949].

The angular momentum of the electromagnetic field can be decomposed into spin angular momentum (SAM) and orbital angular momentum (OAM) [Humblet 1943, Mohammadi et al. 2010]. When a radio beam carrying nonzero angular momentum impinges on an absorbing particle, its angular momentum can be transferred on the particle, thus setting it in rotational motion. This occurs both with SAM and OAM. However, if the particle is not at the beam centre the two angular momenta will give rise to different kinds of rotation of the particle. SAM will give rise to a rotation of the particle around its own centre, that is, to a particle spinning. OAM, instead, will generate a revolution of the particle around the beam axis.

A simple analogy of radio beam carrying both the linear and angular momenta is Earth-Sun system. Radiation of linear momentum corresponds to the Earth-Sun system travelling in space in direction perpendicular to plane determined by the Earth orbit. Spin angular momentum corresponds to Earth rotating around its axis and producing the day-night cycle. Orbital angular momentum, on the other hand, corresponds to Earth rotating around the Sun and producing the seasons.

The classical manifestation of spin angular momentum is circular polarization of a radio wave. On the other hand, the orbital angular momentum of radio beam is a lesser known phenomenon. The main reason is the plane wave approximation in the far field, which is commonly used in many textbooks on electromagnetic theory. By definition, plane waves do not carry orbital angular momentum. Thus, in standard wireless communications we influence either the linear momentum (amplitude) or the spin angular momentum (polarization state) of the radio wave. It is only recently that the orbital angular momentum has found practical use, and that mainly in optical communications [Allen et al. 1992, Gibson et al. 2004, Wang et al. 2012].

All electromagnetic waves are solutions to the general wave equation, which in vacuum can be given as [Andrews & Babiker 2012, page 9]

$$\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}+\frac{\partial^2 E}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}=0$$

$$\frac{\partial^2 B}{\partial x^2}+\frac{\partial^2 B}{\partial y^2}+\frac{\partial^2 B}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}=0$$

where E and B denote, respectively, electric and magnetic field components, c is speed of light in vacuum, and t denotes time.

Without loss of generality, one can consider only the electric field component and assume that the electromagnetic wave propagates along z axis. Thus, one possible solution of general wave equation is [Andrews & Babiker 2012, page 9]

$$E(x,y,z) = A(x,y,z)\exp[-i(\omega t - kz)]$$

where $A(x,y,z)$ denotes the amplitude of the electric field, i denotes imaginary unit, ω is angular frequency, and $k=\omega/c$.

In optics, electromagnetic waves generated by lasers are special types of electromagnetic waves. A laser beam has a characteristic width w, the dimension of the field transverse to the main propagation axis, and a characteristic length L which is some local length along the propagation axis over which the beam characteristics do not vary much. By definition, for a beam w is typically small and L is large in comparison, so that w/L can be considered small. In other words, for a beam one can assume that the amplitude $A(x,y,z)$ varies slowly with z, and thus can neglect the term $\partial^2 E/\partial z^2$ in general wave equation. The resulting equation [Andrews & Babiker 2012, page 12]

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + 2ik\frac{\partial^2 A}{\partial \mathrm{z}} = 0$$

is called paraxial wave equation. Any $A(x,y,z)$ that solves the paraxial wave equation represents a paraxial beam shape. The simplest solutions to paraxial wave equations are plane waves where $A(x,y,z)$ is constant for all x,y, and z. Another simple solutions to paraxial wave equations are spherical waves where $A(x,y,z)$ is inversely proportional to the distance from the wave source $\sqrt{x^2+y^2+z^2}$.

Angular momentum density j of an electromagnetic wave is defined as [Andrews & Babiker 2012, page 6]

$$\vec{j} = \vec{r}\times\vec{p} = \varepsilon_0\vec{r}\times\left(\vec{E}\times\vec{B}\right) = \frac{\varepsilon_0}{c^2}\vec{r}\times\vec{S}$$

where the symbol × denotes vector cross product, $\vec{r}$ denotes the position from the origin, ε_0 is dielectric permittivity of vacuum, and $\vec{S} = \vec{E}\times\vec{B}$ is Poynting vector. By definition, plane electromagnetic waves do not carry orbital angular momentum [Allen et al. 1992], because for a plane wave, the Poynting vector $\vec{S}$ is constant and parallel to the propagation direction at all points on the plane. Thus, one needs a beam for carrying angular momentum.

The earliest work on the orbital angular momentum of light beams took the Laguerre-Gaussian mode as the most easily available source of light possessing an orbital angular momentum. For Laguerre-Gaussian mode the amplitude of the electrical field in cylindrical coordinates (r,φ,z) is [Andrews & Babiker 2012, page 15]

$$\begin{aligned} A(r,\varphi,z) \quad &= \quad \sqrt{\frac{2p!}{\pi(|l|+p)!}}\frac{1}{w(z)}\left[\frac{\sqrt{2}r}{w(z)}\right]^{|l|} L_p^{(|l|)}\left[\frac{2r^2}{w^2(z)}\right]\exp\left[-\frac{r^2}{w^2(z)}\right] \\ &\cdot \quad \exp\left[-\frac{ikr^2z}{2(z^2+z_R^2)} + i(2p+|l|+1)\arctan\left(\frac{z}{z_R}\right)\right]\exp(il\varphi) \end{aligned}$$

where p is the radical mode index, l is the azimuthal mode index, z_R is the Rayleigh range, $w(z)$ is the radius of the beam, and $L_p^{(l)}$ denote associated Laguerre polynomials. The radius of the beam is defined as [Andrews & Babiker 2012, page 13]

$$w(z) = w_0\sqrt{1+(z/z_R)^2}$$

where w_0 is called the beam waist. For circular apertures, with a denoting the radius of the aperture, the beam waist is [Tamburini et al. 2012]

$$w_0 \approx \frac{2a}{1.22\pi}$$

The Rayleigh range z_R is a point where $w(z) = \sqrt{2}w_0$.This a turning point in the propagation of the beam as the beam spot makes the transition from being nearly constant to increasing linearly. For distances shorter than the ray description of the propagation of the electromagnetic wave breaks down and wave description has to be used. In other words, the Rayleigh range determines the turning point between ray-optics and wave-optics [Andrews & Babiker 2012, page 14].

The phase front is defined as the surface that contains all points of the wave that carry the same phase. If the correction due to the radius of curvature of the phase front is neglected, that is, when z=0, then it can be easily shown that the phase front has the form

$$\psi(r,\varphi,z) = kz + l\varphi$$

In other words, the points of constant phase form a helix as shown in Fig. 1. In the case of a beam with helical phase front, the linear momentum, which is perpendicular to the phase front, has an axial component. Thus, the linear momentum in a helical phase front has a component along the propagation direction z, a component along the radical direction r if the beam is expanding, and a component along the azimuthal direction φ. As a consequence, the angular momentum has a nonzero component along the propagation direction z.

The Laguerre-Gaussian beams are not the only examples of beams with helical phase fronts. Bessel beams [McCloin & Dholakia 2005], Mathieu beams [Guttierez-Vega et al. 2000], and Ince-Gaussian beams [Bandres & Guttierez-Vega 2004] can also carry orbital angular momentum [Andrews & Babiker 2012, Chapter 1].

The first use of wireless radio transmission using orbital angular momentum was publicly demonstrated in “waves-on-waves” experiment in Venice on 24 June 2011. The results of the experiment are reported in [Tamburini et al. 2012]. The researchers from University of Padua and Uppsala University showed experimentally, in a real-world setting, that it is possible to use two beams of incoherent radio waves, transmitted on the same frequency but encoded in two different orbital angular momentum states, to simultaneously transmit two independent data streams. In particular, one radio beam was a simple beam without OAM modulation and the other one employed a first-order Laguerre-Gaussian beam obtained with a spiral-phase-plate antenna [Tamburini et al. 2012].

Interception of OAM-based radio signals is problematic because the interceptor would need to know not only traditional transmission parameters such as carrier frequency or transmission time but also precise location of transmitter and peculiar spatial phase distribution of OAM state that was used as a reference pattern to generate the OAM-based radio beam [Gibson 2004].

4. Results and discussion

The authors of [Mohammadi et al. 2010] are criticized in [Edfors & Johansson 2012] for their claims of possible gains in link capacity and general applicability of transmission using radio beams carrying OAM. Some doubts related to OAM-based radios are also presented in [Cartlidge 2012], [Hellemans 2012], [Tamagnone et al. 2012], and [Tamagnone et al. 2013]. We have carefully studied the arguments and counterarguments of both research groups.

A common theory for studying radio communications is based on spherical wave functions and associated transmission, reception, scattering and translation matrices [Hansen 1988]. According to the theory, the electromagnetic field radiated by a transmitter can be completely described by an expansion in a finite number of radiating spherical wave functions. At the receiver end, the same field can be completely described by an expansion in a finite number of local spherical wave functions. The coefficients of the two expansions can be linked through a translation matrix. The rank of the translation matrix equals the theoretical upper bound of available communication channels [Nordebo et al. 2006]. As the distance between the transmitter and the receiver grows, the rank can be seen to approach the value of two [Edfors & Johansson 2012]. These two remaining channels correspond to two orthogonal polarizations. This result would imply limitations for the number of available channels in long distance (far field) radio communications.

We speculate that the disagreement between some research groups on what OAM-based radio is comes from misunderstanding of the system model. Multiple-input multiple-output systems and theory of spherical wave functions use matrix, and thus linear, representation to describe transmitted and received signals. Ordinary electromagnetic wave with planar or spherical phase front cannot carry orbital angular momentum but it can carry spin angular momentum because the wave can be circularly polarized. It can be easily shown that circularly polarized electromagnetic wave can be obtained by superposition, or in other words linear combination, of two linearly polarized electromagnetic waves. Thus, there is no disagreement between theory of spherical wave functions and theory of spin angular momenta.

A key observation in favour of theory partially developed by [Thide 2007, Tamburini et al. 2012] is the fact that electromagnetic waves with helical phase fronts can carry orbital angular momentum. We would like to stress here that the presence of azimuthal dependence of phase $\exp(il\varphi)$ in helical beams makes the communication system nonlinear. Specifically, we conjecture that beams with helical phase fronts can neither be represented as superposition of linearly polarized waves nor they should be treated as linear combination of electromagnetic waves. It is simply impossible to obtain an electromagnetic wave with azimuthal dependence of phase $\exp(il\varphi)$ by summing a finite number of linearly or circularly polarized waves. We believe that common theories used to describe propagation of radio signals break down when one deals with helical beams.

Electromagnetic waves with helical beams are characterized by phase singularity in the centre of the beam as shown in Figs. 2 and 3. In particular, the phase of the signal is undetermined at the centre of the beam where intensity of electromagnetic field is nearly zero. The field intensity of Laguerre-Gaussian modes has a characteristic “donut” shape with most of the energy concentrated in a ring. Consequently, as suggested in [Tamagnone et al. 2012, Tamagnone et al. 2013], the receiver antennas must be positioned in such a way that they are located within the zone of maximum field intensity. In other words, because of the beam spreading, for the optimal reception, the distance between receiver antennas should be a function of the distance between transmitter and receiver. It implies that it can problematic to apply the concept of OAM-based radio in broadcasting applications where the signal is transmitted to several users that are located at different distances from the access point or base station. Nevertheless, OAM-based radios can be used in fixed links where the location of transmitting and receiving stations is perfectly known.

The paper [Edfors & Johansson 2012] has been submitted on 24 February 2011 and accepted for publication on 26 May 2011, that is, before “waves-on-waves” experiment in Venice was conducted. Nevertheless, [Edfors & Johansson 2012], and discussions in [Cartlidge 2012, Hellemans 2012, Tamagnone et al. 2012, Tamagnone et al. 2013] present an interesting point, that is, they demonstrate that in some specific cases, transmission using OAM-based radios can be studied within the framework of multiple antenna systems, but perhaps with more esoteric antennas [Brand 1998, Thide 2007, Deng et al. 2013, Tennant & Allen 2012].

5. Conclusions

In our opinion, orbital-angular-momentum-based radios are perfect communication platforms to construct secure military communication systems. In this work, we are not concerned with link capacity gains or network capacity gains, but we study the use of OAM-based radios to create secure links. So, in our opinion, many of the arguments against OAM-based radios listed in [Edfors & Johansson 2012, Cartlidge 2012, Hellemans 2012, Tamagnone et al. 2012, Tamagnone et al. 2013] do not apply here. Nevertheless, they should be carefully addressed because they can have possible impact on system design.

As the “waves-on-waves” experiment suggests, traditional radio transmissions using linear momentum or spin angular momentum do not interfere with transmissions using orbital angular momentum. Thus, we conjecture that wireless transmissions using OAM are difficult to intercept and very resilient to jamming.

We have so far studied theoretically the properties of OAM-based radio beams. We will study and develop various ways to generate and receive radio beams carrying OAM using, for example, planar, circular, or spiral antenna arrays. New signal processing algorithms for controlling antenna arrays will be developed.

In [Linturi et al. 2013], OAM-based transmission in radio frequency bands has recently been recognized as one of the 50 most important future technologies for Finland.

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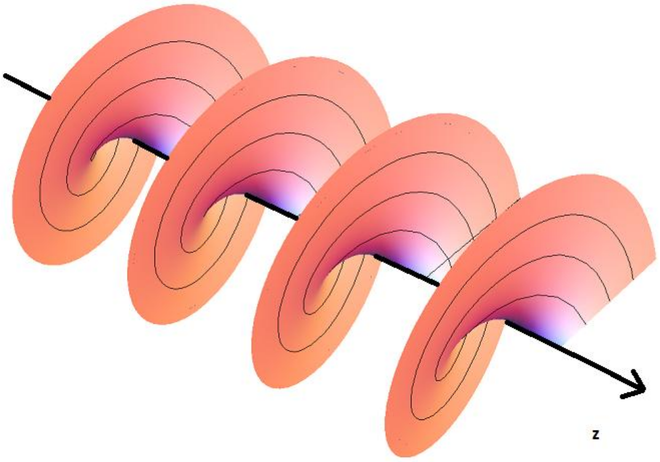

Fig. 1. Helical phase structure of the Laguerre-Gaussian beam associated with a phase factor exp(ilφ) for l=1.

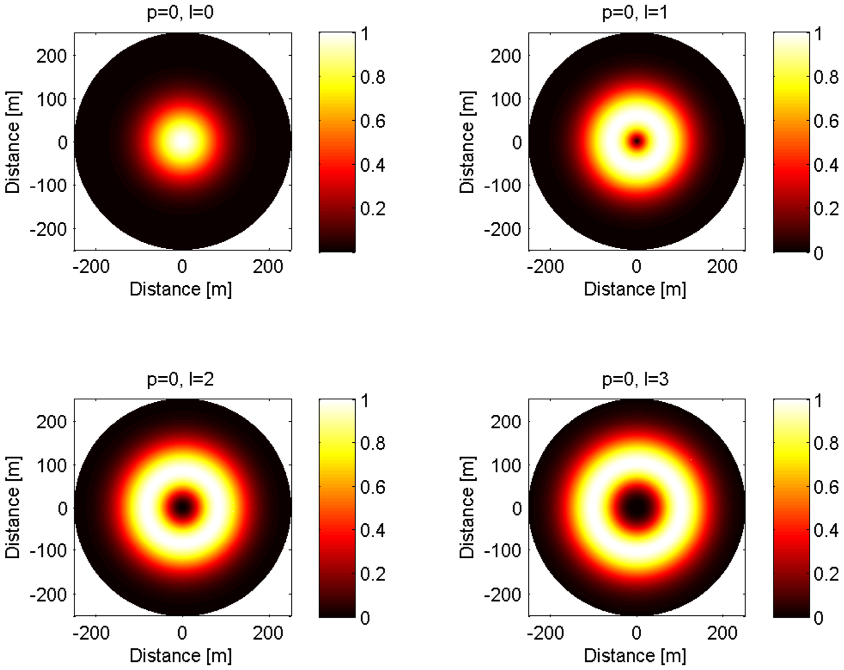

Fig. 2. Relative intensity profiles of various Laguerre-Gaussian modes of radio beam.

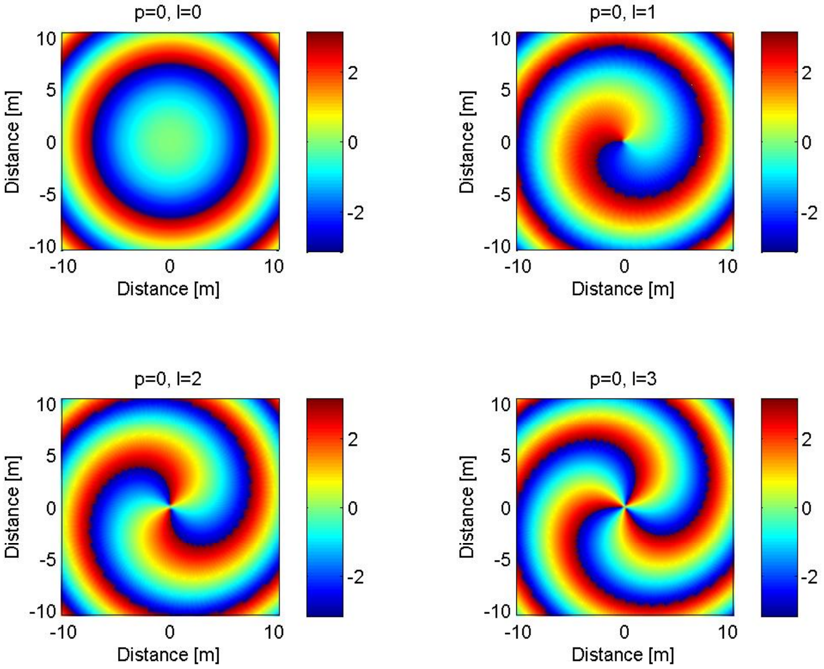

Fig. 3. Phase diagrams of various Laguerre-Gaussian modes of radio beam.