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SUMMARY REPORT

# Application of the theory of rational decision-making to adaptive transmission (RATIONALE)

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**Abstract**
In this work, we study adaptive ad hoc, mesh, and cognitive networks for military use. We show the analogies between decision-theoretic problems and information-theoretic problems in adaptive transmission systems and use this information in designing practical adaptive algorithms. The transmitted energy is known to be a basic system resource. In the case of adaptive transmission, the average transmitted energy should in general be used instead of the average received energy in fair comparisons. We show that the proper normalization of the mathematical model and the selection of the correct performance measure are of critical importance in comparative performance analysis of adaptive transmission systems. We derive a novel fast-simulated diversity model with a sum-of-sinusoids fading channel model and verify both of them with analysis and simulations. We develop a method for performance evaluation using the rational decision theory and the developed models and use it in power control simulations. The simulation results are compared with the analytical results. The optimal scheme based on the decision-theoretic metric differs from the traditional capacity maximization scheme because the proposed metric takes transmission related risks into account. Finally, several practical adaptive power control methods are simulated and their performance analyzed. Our proposed filtered-x least mean square (FxLMS) algorithm is shown to be a generalization of practical algorithms.

## 1. Introduction

Collection of wireless nodes forming ad hoc communication systems without any infrastructure has been interesting in many military applications. There is presently a high interest also in wireless mesh and relay networks, which can be seen as hybrid forms of conventional cellular networks and ad hoc networks. Hybrid networks are more cost-effective and they have a better performance than ad hoc networks. Compared to cellular networks, they are more flexible and scalable. Moreover, there are new generalized adaptive radios called cognitive radios (CRs) that are using various sensors to estimate the status of the environment and making intelligent decisions based on the obtained information. The aim of CRs is to improve the performance of the network; especially the efficiency of spectrum use has been under study.

The wireless channel makes the design of wireless communication systems extremely difficult because the channel characteristics change over time in an unpredictable way. Adaptive transmission techniques are currently considered a very promising method to cope with time-varying effects, improve transmission reliability and maximize the use of available resources. In adaptive transmission systems, the transmitter adjusts the transmission parameters, for example transmission power, data rate, coding scheme, or any combination of those, to the actual state of communication channel in a way that the performance criterion is eventually satisfied. The adaptive control rule that governs the selection of transmission parameters with respect to various transmission constraints is usually called an adaptive transmission strategy.

If the stochastic process representing random fading of the transmission channel is stationary and ergodic and the length of codeword is large enough to reveal ergodic properties of the channel, the channel is information-stable. Wireless communication channels are usually not information-stable channels. Numerous studies on the adaptive transmission in information-unstable channels have been conducted using expected value of mutual information that measures the mutual dependence of the two variables or information outage probability as relevant performance indicators. The information outage probability is defined as the probability that the instantaneous mutual information of the channel is below the transmitted code rate. Outage probability can also be defined as the probability that the channel gain drops below a specified threshold. However, it is unknown whether these performance indicators, although intuitively pleasing, are indeed the best ones because they are usually introduced without rigorous justification.

A fundamental problem in the design of adaptive system which operates in a nonergodic channel is the choice of the preferred probability distribution and the corresponding transmission strategy. The determination of the best choice among several alternatives each of which leads to uncertain possibilities is the subject of rational decision-making under uncertainty that is well developed within the framework of financial mathematics. A new framework for analysing the performance of adaptive transmission in nonergodic channels was presented by Adrian Kotelba[1] in 2008 by exploring analogies between communications theory and finance theory. The novel approach considers jointly reward and risk provided by the adaptive transmission and formulation of the performance measure as a certain risk-reward ratio. The risk-reward approach is used to quantitatively describe the performance of power control schemes. A power control is defined to be efficient if it minimizes the risk for a given level of reward or maximizes the reward for a given level of risk. The set of all efficient power control schemes sets the fundamental limit, called *efficient frontier*, for the performance because no other scheme can be constructed that achieves the performance above the frontier. Adaptive power control rule achieving the fundamental limit has been analytically derived.

## 2. Research objectives and accomplishment plan

The question we address in this study is how to determine efficient practical adaptive transmission strategies achieving almost optimal trade-off between average transmission data rate and the risk associated with that particular data rate for cognitive ad hoc and mesh network scenarios for military communication. In addition, what are appropriate performance indicators that objectively and meaningfully describe the performance of adaptive systems in those situations? The problem will be studied in nonergodic channels, e.g., slowly fading channels resulting from slow moving nodes.

The research plan has been divided into three phases including 1) System requirements and system specification, 2) Specification of algorithms, and 3) Focused demonstrator. Requirements include performance and complexity requirements in terms of the efficiency of the use of materials, energy, and control information. This has been done during the first year of the project as well as part of the algorithm work. Phase 2) has been considered during the second year and algorithms have been developed and tested using the rational decision theory based performance metric. The development of the algorithms has been made with analysis and MATLAB simulations.

[1] *A. Kotelba and A. Mämmelä, " Application of financial risk-reward theory to adaptive transmission," in Proc. VTC-Spring, pp. 1756– 1760, May 2008.*

The third phase will be covered in the third year, if funding will be received. As proposed by the military expert, interference issues will be included in the algorithm development work. Regarding the demonstrator, operator visualization tools are developed to help to understand the complexity and status of the system. The proposed demonstrator will be implemented in software to minimize the costs. The simulation times need to be minimized.

# 3. Materials and methods

## 3.1 Channel model

Wireless links can be considered within the framework of individual rationality used in the decision theory. We consider a nonergodic slowly varying channel, corresponding to low mobility that can be modelled using the Doppler power spectrum. The rate of the channel variation, i.e., the effect of mobility, can be characterized by Doppler frequency $f_d$. A flat Doppler power spectrum corresponds to urban environments, where the transmitter is set above rooftop level, and indoor environments. Thus, the time-variant channel gain is written using a sum of complex exponentials as

$$h[k] = a \sum_{i=1}^{N} e^{j(2\pi f_i k + \phi_i)} \quad (1)$$

where $N$ is the number of multipath components having essentially the same delay, $a$ is the amplitude of every complex exponential, $f_i$ is the Doppler shift of the $i$th component, $\phi_i$ is the random phase shift of the $i$th component uniformly distributed in range $[0, 2\pi[$ and $k$ is time.

If the Doppler shifts of complex exponentials are equally spaced between $[-f_d, f_d]$, the channel gain becomes periodic in time. Periodicity can be removed if the shifts are properly chosen to make the channel gain quasi-periodic. The Doppler shift range is divided into $N$ equal-size parts. The frequencies of the components differ a random uniformly distributed amount from the equal space solution. With these selections, we obtain the whole spectrum range to use in every realization of the channel. The spectrum is made symmetric over zero frequency, which makes the autocorrelation function of the channel real. This selection makes simulations faster.

Major approaches for normalization of the channel include normalization of the average energy gain or the peak energy gain to unity, i.e., average and peak normalization, respectively. When peak normalization is used, $a = 1/N$ in (1). If average normalization is used, $a = 1/\sqrt{N}$.

In a diversity system the transmitter power control algorithm should control the power of the diversity combiner output in the receiver. The model for simulating diversity is shown in Fig. 1. The proposed model is equivalent with the analysis done in famous book of Proakis[2]. Data can be added as shown when no intersymbol interference (ISI) is present and the system is coherent. ISI would make the noise correlated at separate sampling instants and thus the model would not be applicable. We assume linear, slowly fading, and frequency nonselective channel. The model can represent also frequency-selectivity in such Rake systems where there is no significant ISI or interpath interference.

[2] J. G. Proakis, Digital Communications, 4th ed. McGraw-Hill, 2001.

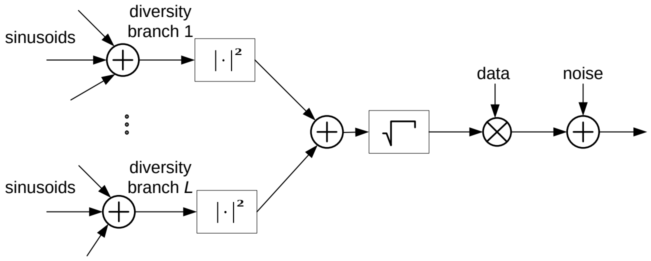


Fig 1. Simulation model for diversity, coherent system.

### 3.2 Rational decision theory concepts

In finance theory the *reward* is measured as an expected return $r$ on investment in excess of some predefined threshold $t$. In adaptive transmission, the return is the actual link spectral efficiency $r$ for certain energy "investment". The reward $d$ is the difference between the expected value of the link spectral efficiency $\mu_r = E[r]$, and the target link spectral efficiency $t$, $d = E[r - t] = \mu_r - t$.

The notion of *risk* is applied in economics as a property of uncertain options or lotteries, which affects decision-making. In adaptive transmission the risk can be measured with the $n$th root of the $n$th order lower partial moment of the link spectral efficiency distribution $p_r(r)$

$$l_n^-(t) = \int_{-\infty}^{t} (t-r)^n p_r(r)dr, \quad n \geq 0. \tag{2}$$

When $n = 0$, we have the outage probability. A risk measure with value $n = 1$ is called expected shortfall. This meter is commonly used in nuclear engineering. In addition to the probability, the metric includes the magnitude of the potential shortfall. However, large infrequent losses represent the same risk as small and frequent losses. When $n > 1$, user's risk aversion can be used. The risk aversion means that a rational user, when offered several power control schemes with the same expected link spectral efficiency, prefers the scheme with the lowest risk. For analytical tractability, $n = 2$ is used. The second order partial moment $l_2^-(t)$ is known as the below-target semivariance.

Basically, the proposed risk measure defines how far we are from the desired value. When the probability of outage is used as a performance criterion, we assume that the performance of the link is good enough above the certain threshold and not working at all below the threshold. Proposed risk measure (2) defines how badly fading affects the performance in a smoother way. The usability worsens when the distance to the desired value increases. Video transmission is a good example of the phenomenon. The quality of the video becomes worse but still something can be seen and understood.

In decision theory, and especially in mean-risk models, the term "efficiency" refers to the *optimal trade-off* between mean performance and risk associated with a given mean performance. A trade-off between reward and risk is measured quantitatively by a reward-to-semivariability ratio. The most general reward-to-semivariability ratio is Kappa ratio, which is defined as the ratio of the reward to the $n$th root of the $n$th order partial moment,

$$\kappa_n(t) = \frac{E[r-t]}{\sqrt[n]{l_n^-(t)}}, \quad n > 0. \tag{3}$$

The optimal scheme is the one with the highest reward-to-semivariability ratio $\kappa_n(t)$ since it maximizes the reward per unit of risk taken. The optimal combinations of mean performance and risk are called efficient combinations. The efficient frontier is the fundamental limit of the mean-risk performance because no other scheme can be constructed that achieves the performance above the efficient frontier.

## 3.3 Adaptive power control methods

### 3.3.1 *Theoretical methods*

Power control algorithms can, in general, be divided into water filling and truncated channel inversion (TCI). If water filling is used, the transmitted energy is $E_{\mathrm{tx}} = \bar{E}_{\mathrm{tx}}(1/\gamma_0 - 1/\gamma_H)$ for $\gamma_H \geq \gamma_0$ and zero otherwise where the quality of the channel is defined as $\gamma_H = \bar{E}_{\mathrm{tx}}|H|^2 / N_0$ and $\gamma_0$ is a cut-off value, which is found by a numerical method. $\bar{E}_{\mathrm{tx}}$ is average transmitted energy per symbol, and $|H|^2$ is the instantaneous energy gain of the channel. If truncated channel inversion is used, the transmitted energy is

$$E_{\mathrm{tx}} = \bar{E}_{\mathrm{tx}}(\sigma_0 / \gamma_{\mathrm{H}}) \quad (4)$$

for $\gamma_{\mathrm{H}} \geq \gamma_0$ and zero otherwise where $\sigma_0$ is a constant selected so that the average transmitted energy is $\bar{E}_{\mathrm{tx}}$. The cut-off value is found by a numerical method. The cut-off value is $\gamma_0 = 0$ for full channel inversion, which can be used if $|H|^2$ is never exactly zero. $\bar{E}$ denotes expectation of a random variable $E$. Basically the difference between the described two approaches is that the water filling allocates more power to the better channel states whereas channel inversion aims at maintaining the desired signal strength at the receiver by inverting the channel power gain based on the channel estimates. Several practical approaches can be called inverse power control approaches.

FxLMS algorithm

Filtered-x least-mean-square (FxLMS) algorithm was proposed for adaptive inverse power control by us in 2007. The algorithm updates the coefficient $c_k$ of a one-tap filter as

$$c_k = c_{k-1} + \mu x_k^{'} \varepsilon_k \quad (5)$$

where $\mu$ is the adaptation step size of the algorithm, the filtered input signal is $x_k^{'} = |x_k \hat{h}_k|$, $\hat{h}_k$ is the estimated instantaneous channel gain, and $\varepsilon_k$ is the error signal to be minimized. The optimal step size with an estimated channel gain is given by

$$\mu_{\mathrm{opt}} = \frac{1}{(|x_k|^2 |\hat{h}_k|^2) + c_{\mathrm{term}}} \quad (6)$$

where $c_{\mathrm{term}}$ is a small number that prevents the adaptation step size to grow to infinity when the estimated received power is very small. When we are using the FxLMS algorithm for power control, we can reduce the complexity of the transmitter by doing as much as possible calculations at the receiver. This reduces also information in the feedback channel since only the signal $c_{k-1}$ is needed to be sent to the transmitter. It can be further coded to remove the redundancy since the consequtive values of $c_{k-1}$ are highly correlated in a slowly fading channel.

### 3.3.2 *Fixed and variable step adjustment power control*

Typically the power control time interval in a code division multiple access (CDMA) system is around 1 ms. CDMA power control employs both closed and open loop methods; we restrict our investigation purely on the closed loop part and use the same 1 ms interval. The base station measures the signal-to-interference ratio (SIR) or the average received power over *m* symbols and compares it to a reference power level $P_{\text{ref}}$. As a result of the comparison the base station tells the mobile station to adjust its transmission power upwards or downwards by a control step size $\Delta P$.

*A p*ractical fixed-step adjustment power control (FSAPC) method uses 1 dB steps. The power control algorithm can be written as

$$P_k = P_{k-1} + C_k \Delta P \text{ [dB]} \tag{7}$$

where the power control command is $C_k = \begin{cases} +1, & \varepsilon_k \geq 0 \\ -1, & \varepsilon_k < 0 \end{cases}$. The weakness of this fixed-step power control method is that it is still too slow for fast moving vehicles since the fading can be tens of dB even every half a carrier wavelength.

Variable step power control methods have been proposed to overcome the weakness of the fixed step solution. The basic idea is that when the power of received signal is far from the desired, the control step is increased to reach the desired level faster. The power control command for variable step adjustment power control (VSAPC) is

$$C_k = \begin{cases} 3, & \text{when } P_{\text{err}} < -5\kappa \\ 2, & -5\kappa \leq P_{\text{err}} < -3\kappa \\ 1, & -3\kappa \leq P_{\text{err}} < -\kappa \\ 0, & -\kappa \leq P_{\text{err}} < \kappa \\ -1, & \kappa \leq P_{\text{err}} < 3\kappa \\ -2, & P_{\text{err}} \geq 3\kappa \end{cases} \tag{8}$$

where $P_{\text{err}}$ is the power of error signal in dB and $\kappa = 0.5\Delta P$. Thus, the control speed with this method is up to 3 dB/power control command.

A recently proposed adaptive closed loop power control (ACLPC) method is described in Long Term Evolution (LTE) requirements. The uplink receiver estimates the signal-to-interference and noise ratio (SINR) of the received signal and compares it with the SINR target value. If the estimated value is below the target, the transmission power is increased. Otherwise, the transmitted power will be decreased. The transmitter power control (TPC) command is checked in every subframe whose duration is 1 ms. Thus, we use the same model as previously to see the performance of the closed loop part. Power control command $C_k$ values are $C_k$ = {–4, –1, 1, 4} (dB), which means that only two bits are needed for TPC command.

## 4. Results and discussion

The probability density function of the received SNR value using the proposed diversity model is shown in Fig. 2. Both analytical and simulated results are presented and shown to match well, thus verifying the applicability of the diversity model as well as the channel model because the diversity channels are implemented using the proposed sum-of-complex exponentials model.

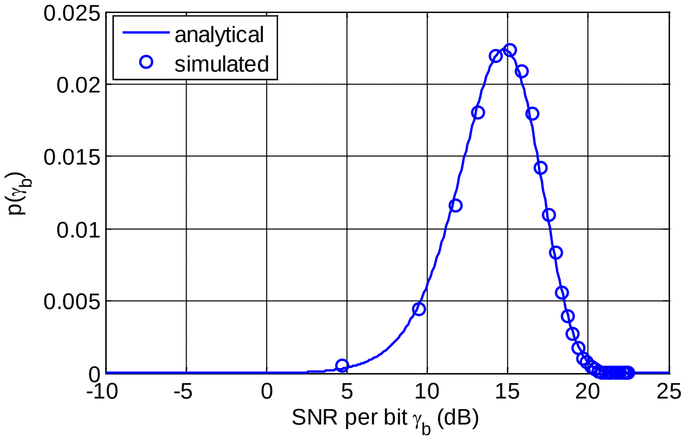


Fig 2. Probability density function of SNR for diversity system.

We considered both single-input single-output (SISO) and diversity channels in TCI simulations. Maximal ratio combining (MRC) diversity ($L$ = 2 and $L$ = 4) was considered in diversity experiments. Both analytical results using a Rayleigh fading channel, and simulated results are shown in Fig. 3.

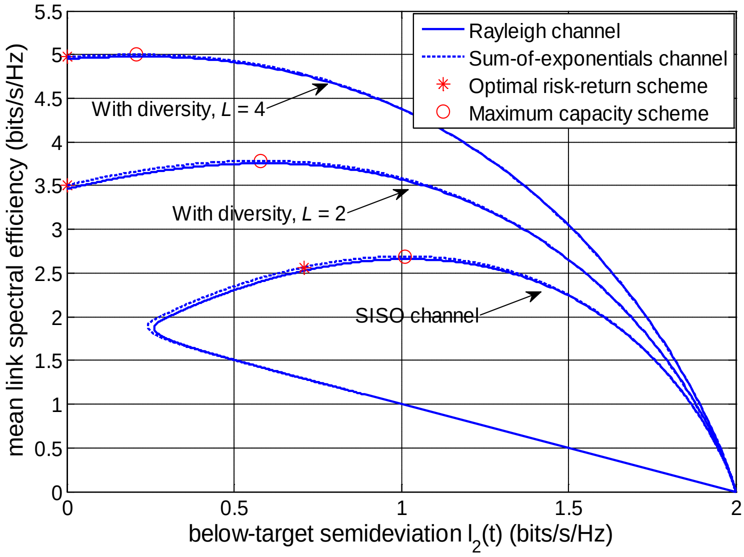


Fig 3. Risk-return curves of truncated channel inversion over SISO an diversity channels.

**Target link spectral efficiency for the experiments was set to $t$ = 2 bits/s/Hz.** When the SISO channel is considered, there is a clear turning point in the risk-return curve. The capacity of the Rayleigh fading channel with the total channel inversion is zero which means that the risk is very high. The risk is also high with the sum-of-complex exponentials channel but the capacity is not zero. The high risk comes from the fact that without any cut-off value the transmitted signal has to be transmitted also during the deepest fades in the channel. Total channel inversion can be used only with channels having $E[1/|h|^2] < \infty$ which is not valid for the Rayleigh channel. The sum-of-complex exponentials channel achieves slightly better risk-reward performance since the model does not include a zero gain and the peak gain is limited. In the following figures, only the sum-of-complex exponentials channel is considered.

When we start to increase the cut-off value from the zero in the SISO channel, the below-target semideviation reduces and at the same time the link spectral efficiency increases. The risk reduces since the power is not wasted in the deepest fades. After a certain minimum-risk point, the risk starts to increase again because the probability of outage increases. The risk curve for all the cases can be seen in Fig. 4. The link spectral efficiency increases further when the cut-off value is increased until the maximum capacity scheme is achieved and after that increasing the outage probability starts to reduce the return as seen in Fig. 4. With very high cut-off values the risk is again very high and the link spectral efficiency approaches zero. When diversity is applied, the risk is very small with low cut-off values and thus the risk-return performance is also good. However, the maximum capacity point is achieved with a higher cut-off value than in the SISO channel.

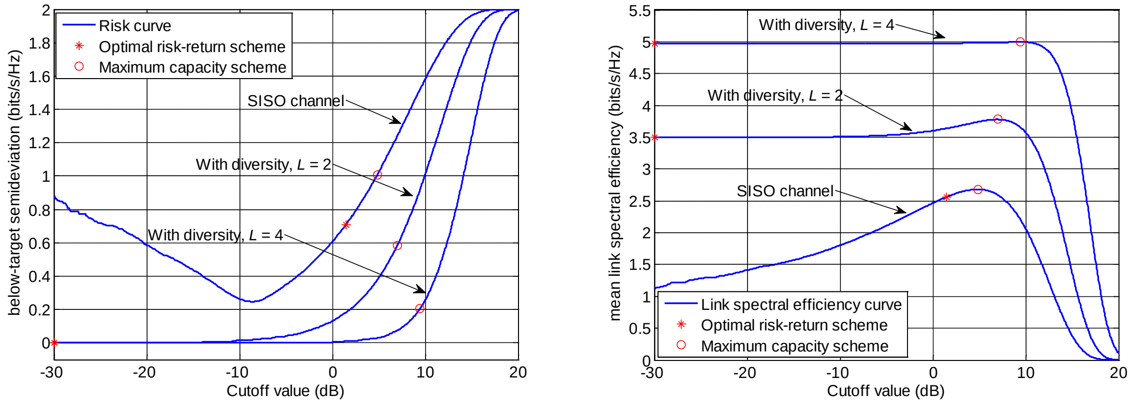


Fig 4. Risk curves and return curves of truncated channel inversion over SISO an diversity channels.

The traditionally used maximal capacity approach gives different rules for power control than the rational decision theory. Actually, even though the maximum capacity scheme for a diversity channel requires use of the cut-off value, it has been suggested that with diversity the total channel inversion might be a better choice than the truncated channel inversion. The rational decision-making leads unambiguously to this solution.

The results shown above suggest rethinking of optimal way to set thresholds for truncated channel inversion power control. The use of the risk-reward approach gives lower threshold for the SISO channel, leading to a slightly lower link spectral efficiency but also with a lower probability of outage. This means that delays in transmission are shorter. When diversity is applied, the proposed approach leads clearly to the conclusion of using full inversion for transmission, again with a slightly lower reward but with no risk, i.e., without outages. The curves are mostly smooth but with low cut-off values there is a small inaccuracy in SISO channel. This is due to the fact that we are operating with the tail values of the energy distribution. There are fewer samples in these values and thus the estimate of the average obtained with the Monte Carlo method is not so accurate anymore. The problem disappears when diversity is applied.

The risk-reward performance of the practical full inversion power control algorithms cannot be measured using exactly the same method than above. The reason is that an outage has to be defined in a different way and the average transmission power cannot be fixed if the aim is to keep the received SNR adaptively at the target level. We define an outage as a received signal level that is more than 2 dB below the target. Based on Vysochanskij-Petunin inequality and assuming 1 dB standard deviation, roughly 90 percent of cases are closer than 2 dB to the desired value. Now the risk-reward performance is given with one single point for each power control rule. The results are shown in Fig. 6. The result of a simulation with an optimal inversion is also provided and the risk of being 2 dB below the target is shown to be zero. The mean link spectral efficiency with the optimal inversion is slightly better than with the adaptive approaches because it can keep the received SNR high enough also during the deep fades.

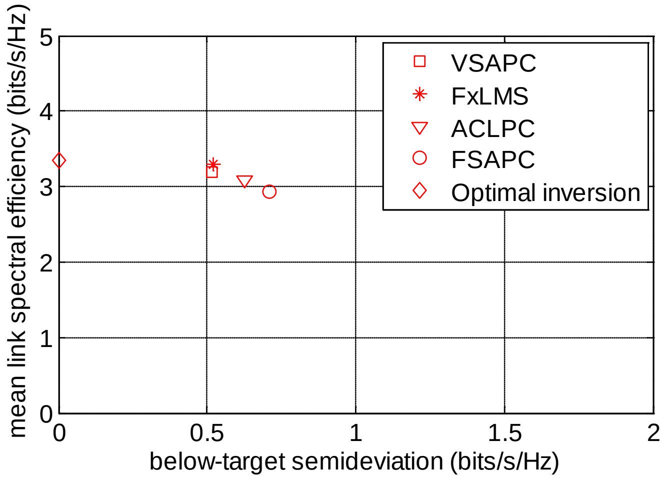


Fig 5. Risk-return performance of practical power control rules in a SISO channel.

The FxLMS method gives the best performance and the FSAPC method is clearly the worst. FxLMS, VSAPC, and ACLPC methods have bigger step sizes which make adaptation faster. This can be seen in the rise times in Table 1. Rise time is the time required for the received signal to change from the initial value, when transmitted signal is 0 dB, to the required 10 dB value in a time-variant channel. The results shown are average values over several simulations. In addition, the system does not spend so much time during the deep fade than with smaller adaptation steps. That is the reason for the better risk performance. Since the FxLMS control is the best in reward and equally good with the VSAPC in risk performance, it achieves the best risk-reward values using the Kappa ratio defined in (3) as a measure. The risk-reward performance difference between methods is very clear when we look at the Kappa ratio values in Table I.

Table I. Kappa ratio values

| | **Rise time (ms)** | **Standard deviation (dB)** | **Kappa ratio** | **Average transmitted SNR (dB)** |
|---|---|---|---|---|
| **FSAPC** | 19 | 1.48 | 1.32 | 25.71 |
| **ACLPC** | 7 | 1.09 | 1.66 | 26.75 |
| **VSAPC** | 9 | 1.04 | 2.30 | 26.68 |
| **FxLMS** | 4 | 1.03 | 2.48 | 26.65 |

There is a problem with the nonfixed average power constraint in the performance comparison since different methods use different amount of transmitted energy for communication. However, the difference is very small between ACLPC, VSAPC, and FxLMS methods as shown in Table 1. Thus, the performance comparison between these methods is pretty fair. FSAPC method suffers since it is spending more time during deep fades with a lower power and consequently the outage is also higher. Standard deviation of the received SNR, averaged in decibel domain, shows clearly the gain of using adaptive step sizes in control. Based on the achieved results, adaptive step sizes are much more preferable to be used in communication. The FxLMS algorithm achieves the best performance with the given fundamental metric.

# 5. Conclusions

We studied application of rational decision theory to optimization and development of adaptive transmission in this project. As a novel aspect the theory enables inclusion of the risk as an additional criterion in the development. We have presented models for a fading channel and diversity channel and verified both of them with analysis and simulations. Power control simulations using the proposed models show the importance of the selection of proper performance criterion and normalization.

The investigated theory and the corresponding performance metric give new insights to the development of adaptive transmission methods. The results shown above suggest rethinking of the optimal way to set thresholds for the truncated channel inversion power control. When diversity is applied, the proposed theory leads to the solution that has been intuitively thought to be the best choice, i.e., no truncation should be used at all. The theory also gives good tools for comparing the performance of different practical adaptive algorithms. Our proposed FxLMS algorithm is shown to achieve the best performance. The algorithm also makes the general investigation of adaptive power control possible.

Compared to the aims set in the beginning of the project, we have clearly achieved them. Several publications have been produced as results of the project. As proposed by a military expert following a project, a fruitful topic to consider in the future is the inclusion of interference to the work. How it affects to the adaptive transmission and to the algorithms considered in this work?

# 6. Scientific publishing and other reports produced by the research project

- A. Mämmelä, A. Kotelba, M. Höyhtyä, and D. P. Taylor, “Relationship of average transmitted and received energies in adaptive transmission,” *IEEE Transactions on Vehicular Technology*, vol. 59, pp. 1257–1268, March 2010. *This journal article discusses the importance of the selection of a proper performance metric in adaptive transmission strategy development. The importance of considering transmitted energy instead of received energy is emphasized. Normalization of the channel and the use of sum-of-sinusoids channel model is discussed.*
- M. Höyhtyä, A. Kotelba, and A. Mämmelä, ”Practical adaptive transmission with respect to rational decision theory,” *VTC-Spring* 2011 conference (submitted). *In this conference paper we study and develop rational decision theory based performance evaluation for adaptive transmission strategy development. The achieved results suggest rethinking of the optimal way to perform adaptive transmission in fading channels. We also show the applicability of the method in ranking several practical algorithms. Our own proposed power control method is seen as a generalization of practical algorithms.*
- A. Mämmelä, A. Kotelba, M. Höyhtyä, and D. P. Taylor, “Link budgets: How much energy is really received,” *Vehicular Technology*, *Intech,* Austria (accepted for publication). *This book chapter includes discussion about the use of rational decision theory and the effect of adaptive power control techniques in link budget analysis.*